

## Calcul des déformations des ressorts hélicoïdaux

### Ressort conique - Force concentrée dans l'axe du ressort

#### Flexion et torsion

##### Fil rond en acier

$$d := 5 \cdot \text{mm} \quad S := \pi \cdot \frac{d^2}{4} \quad E := 2.0 \cdot 10^5 \cdot \text{N} \cdot \text{mm}^{-2} \quad G := \frac{E}{2.5} \quad \rho := 7.85 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

➔ Référence : E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$J_t := J_{t\_circ}(d) \quad I_{22} := I_{f\_circ}(d) \quad I_{33} := I_{22}$$

$$W_t := W_{t\_circ}(d) \quad W'_t := W_t \quad W_{f2} := W_{f\_circ}(d) \quad W_{f3} := W_{f2}$$

#### Caractéristiques du ressort

$$D := 40 \cdot \text{mm} \quad R_0 := 0.5 \cdot D \quad R_1 := 0.1 \cdot D$$

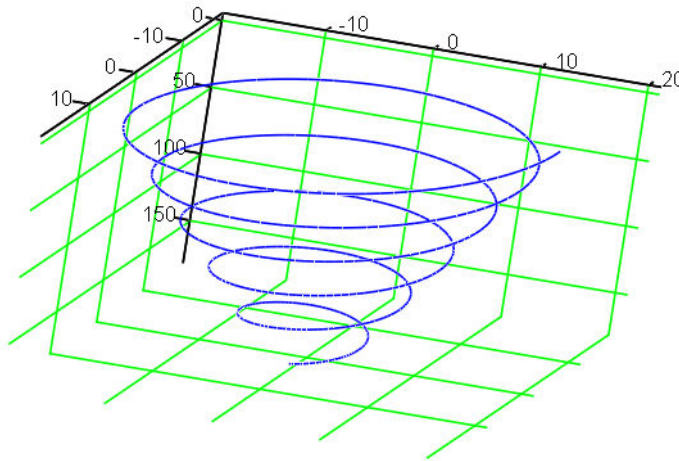
$$\beta_0 := 15 \cdot \text{deg} \quad p := 2 \cdot \pi \cdot R_0 \cdot \tan(\beta_0) \quad n_{sp} := 5.25 \quad \psi_{AB} := 2 \cdot \pi \cdot n_{sp} \quad p = 33.671 \text{ mm}$$

#### Forme du ressort

$$r_0(\alpha) := R_0 - (R_0 - R_1) \cdot \frac{\alpha}{\psi_{AB}} \quad \beta(\alpha) := \arctan\left(\frac{p}{2 \cdot \pi \cdot r_0(\alpha)}\right) \quad s(\alpha) := \int_0^\alpha \frac{r_0(\alpha')}{\cos(\beta(\alpha'))} d\alpha'$$

$$x_0(\alpha) := r_0(\alpha) \cdot \cos(\alpha) \quad y_0(\alpha) := r_0(\alpha) \cdot \sin(\alpha) \quad z_0(\alpha) := \frac{p}{2 \cdot \pi} \cdot \alpha \quad L := s(\psi_{AB}) \quad L = 43.89 \text{ cm}$$

$$n := 100 \cdot n_{sp} + 1 \quad i := 0..n-1 \quad \alpha_{0_i} := \frac{\psi_{AB}}{n-1} \cdot i \quad x_i := x_0(\alpha_{0_i}) \quad y_i := y_0(\alpha_{0_i}) \quad z_i := z_0(\alpha_{0_i})$$



$$\left( \frac{x}{\text{mm}}, \frac{y}{\text{mm}}, \frac{z}{\text{mm}} \right)$$

#### Force concentrée axiale

$$\psi_F := \psi_{AB} \quad \chi_F := 0 \cdot \text{deg}$$

$$P := 400 \cdot \text{N} \quad r_F := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_F := 0 \cdot \text{deg} \quad \gamma_F := 0 \cdot \text{deg}$$

$$\mathbf{F} := P \cdot \begin{pmatrix} \cos(\lambda_F) \cdot \sin(\gamma_F) & \sin(\lambda_F) \cdot \sin(\gamma_F) & \cos(\gamma_F) \end{pmatrix}^T \quad \mathbf{F}^T = (0 \ 0 \ 400) \text{ N}$$

$$C := 0 \cdot \text{N} \cdot \text{m} \quad r_C := z_0(\psi_F) \cdot \cos(\chi_F)^{-1} \quad \lambda_C := 0 \cdot \text{deg} \quad \gamma_C := 0 \cdot \text{deg}$$

$$\mathbf{C} := C \cdot \begin{pmatrix} \cos(\lambda_C) \cdot \sin(\gamma_C) & \sin(\lambda_C) \cdot \sin(\gamma_C) & \cos(\gamma_C) \end{pmatrix}^T \quad \mathbf{C}^T = (0 \ 0 \ 0) \text{ N}$$

➡ Référence :E:\Résonateur (TA)\Ressorts hélicoïdaux\Ressort conique E\_L - F&C.mcd(R)

### Torseur des forces de cohésion

$$\alpha_M := 40 \cdot \text{deg} \quad \mathbf{R}_c(\alpha_M)^T = (0 \quad 0 \quad 400) \text{ N}$$

$$\mathbf{M}_c(\alpha_M)^T = (-5.055 \quad 6.025 \quad 0) \text{ N} \cdot \text{m}$$

### Sollicitations

Traction-compression  $N_c(\alpha_M) = 105.188 \text{ N}$

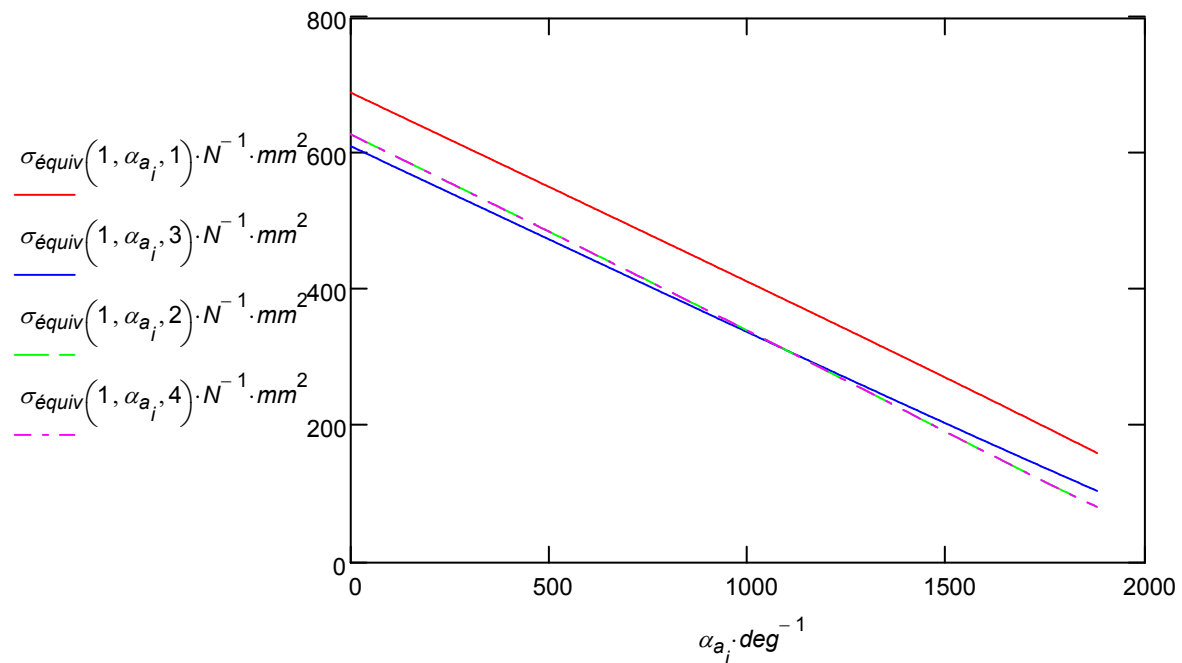
Effort tranchant  $Q_2(\alpha_M) = 0 \text{ m} \cdot \text{kg} \cdot \text{s}^{-2}$   $Q_3(\alpha_M) = 385.922 \text{ m} \cdot \text{kg} \cdot \text{s}^{-2}$

Moment de torsion  $M_t(\alpha_M) = 7.588 \text{ N} \cdot \text{m}$

Moments de flexion  $M_{f2}(\alpha_M) = 0 \text{ N} \cdot \text{m}$   $M_{f3}(\alpha_M) = -2.068 \text{ N} \cdot \text{m}$

### Contraintes

$$n := 201 \quad i := 1 \dots n - 1 \quad \alpha_{a_i} := (i - 1) \cdot \frac{\psi_{AB}}{n - 1}$$



### Calcul des déplacements linéiques

#### Position du déplacement désiré

$$\alpha_M := \psi_{AB}$$

#### Déplacement dans la direction de Ox

$$\lambda := 0 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = -0.017 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 0 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = 0.378 \text{ mm}$$

$$\delta_x(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_x(\alpha_M) = 0.362 \text{ mm}$$

#### Déplacement dans la direction de Oy

$$\lambda := 90 \cdot \text{deg} \quad \gamma := 90 \cdot \text{deg}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 5.526 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 0 \text{ mm}$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = 0.323 \text{ mm}$$

$$\delta_y(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_y(\alpha_M) = 5.849 \text{ mm}$$

Déplacement dans la direction de Oz     $\lambda := 0 \cdot \text{deg}$      $\gamma := 0 \cdot \text{deg}$

$$\delta_{tv}(\alpha_M, \lambda, \gamma) = 6.183 \text{ mm}$$

$$\delta_{fv2}(\alpha_M, \lambda, \gamma) = 0 \text{ mm}$$

$$|\mathbf{v}(\lambda, \gamma)| = 1$$

$$\delta_{fv3}(\alpha_M, \lambda, \gamma) = 0.667 \text{ mm}$$

$$\delta_z(\alpha) := \delta_v(\alpha, \lambda, \gamma)$$

$$\delta_z(\alpha_M) = 6.849 \text{ mm}$$

### Calcul des déplacements angulaires

Déplacement angulaire autour de Ox     $\lambda_c := 0 \cdot \text{deg}$      $\gamma_c := 90 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = -1.787 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0 \text{ deg}$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = -0.099 \text{ deg}$$

$$\theta_x(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_x(\alpha_M) = -1.886 \text{ deg}$$

Déplacement angulaire autour de Oy     $\lambda_c := 90 \cdot \text{deg}$      $\gamma_c := 90 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 0.133 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0 \text{ deg}$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = 0.065 \text{ deg}$$

$$\theta_y(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_y(\alpha_M) = 0.198 \text{ deg}$$

Déplacement angulaire autour de Oz     $\lambda_c := 0 \cdot \text{deg}$      $\gamma_c := 0 \cdot \text{deg}$

$$\theta_{tcv}(\alpha_M, \lambda_c, \gamma_c) = 9 \text{ deg}$$

$$\theta_{fcv2}(\alpha_M, \lambda_c, \gamma_c) = 0 \text{ deg}$$

$$|\mathbf{cv}(\lambda, \gamma)| = 1$$

$$\theta_{fcv3}(\alpha_M, \lambda_c, \gamma_c) = -7.2 \text{ deg}$$

$$\theta_z(\alpha) := \theta_{cv}(\alpha, \lambda_c, \gamma_c)$$

$$\theta_z(\alpha_M) = 1.8 \text{ deg}$$

### Graphe de la déformation

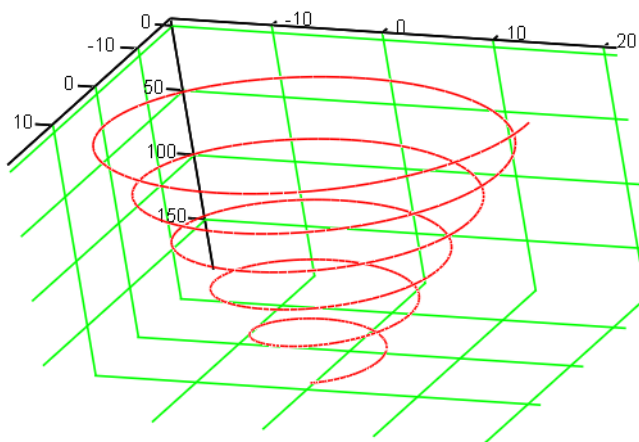
$$x_d(\alpha) := x_0(\alpha) + \delta_x(\alpha)$$

$$y_d(\alpha) := y_0(\alpha) + \delta_y(\alpha)$$

$$z_d(\alpha) := z_0(\alpha) + \delta_z(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \quad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \quad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$$X := \overrightarrow{x_d(\alpha_0)} \quad Y := \overrightarrow{y_d(\alpha_0)} \quad Z := \overrightarrow{z_d(\alpha_0)}$$



$$\left( \frac{X}{\text{mm}}, \frac{Y}{\text{mm}}, \frac{Z}{\text{mm}} \right)$$